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OECD Benchmark for Uncertainty Analysis in Best-Estimate Modelling (UAM) for Design, Operation and Safety Analysis of LWRs - Fifth Workshop (UAM-5)

Burnup Uncertainty Analysis for PWR-HFP Exercise I-1

Authors: O. Cabellos, C. Ceresio, J. S. Martinez, C.J. Diez

Department of Nuclear Engineering

(Polytechnical University of Madrid, UPM)

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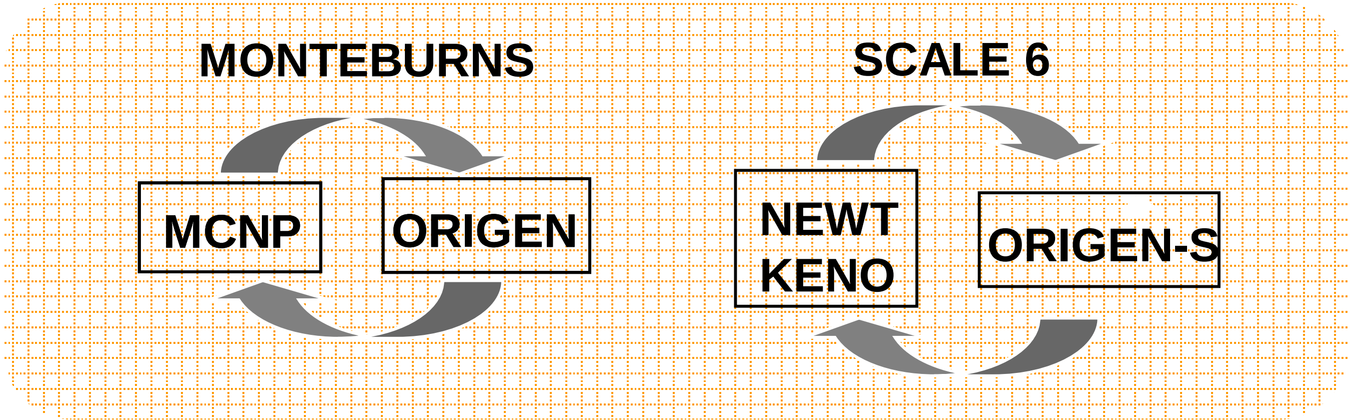
Propagation of uncertainties in burn-up calculations

Burn-up credit analyses are based on depletion calculations that provide an accurate prediction of spent fuel isotopic contents, followed by criticality calculations to assess keff

Different systems coupling a neutron transport code with an isotopic inventory code are being applied:



Need of evaluating uncertainties in isotopics for spent fuel and assess their potential impact on reactivity:

- assumptions made in the calculation models, coupling, …
- uncertainties in nuclear data: cross section, fission yields and decay data

Sources of uncertainties in a depletion calculation

The influence of all these sources should be investigated in order to understand and quantify the uncertainties associated with computer code predictions for spent fuel isotopics:

$$\frac{dN}{dt} = [\lambda]N + \left[\sigma^{eff}\right] \cdot \Phi N + \left[(\gamma\sigma_{fiss})^{eff}\right] \cdot \Phi N = A \cdot N$$

$$N = N\left(\lambda, \sigma^{eff}, \Phi\right) = N(\lambda, \gamma, \sigma^{g}, \phi^{g}(E), \Phi)$$

- Uncertainties in decay constants: Δ_{λ}
- Uncertainties in one-group effective xs: $\Delta_{\sigma^{eff}}$

$$\sigma^{eff} = \sum_{g} \sigma^{g} \phi^{g} \Big/ \sum_{g} \phi^{g}$$

- uncertainties in the evaluated nuclear xs data: $\Delta\sigma^{g}$

- uncertainties in the flux spectrum obtained from the transport calculation: $\Delta\phi^{g}$

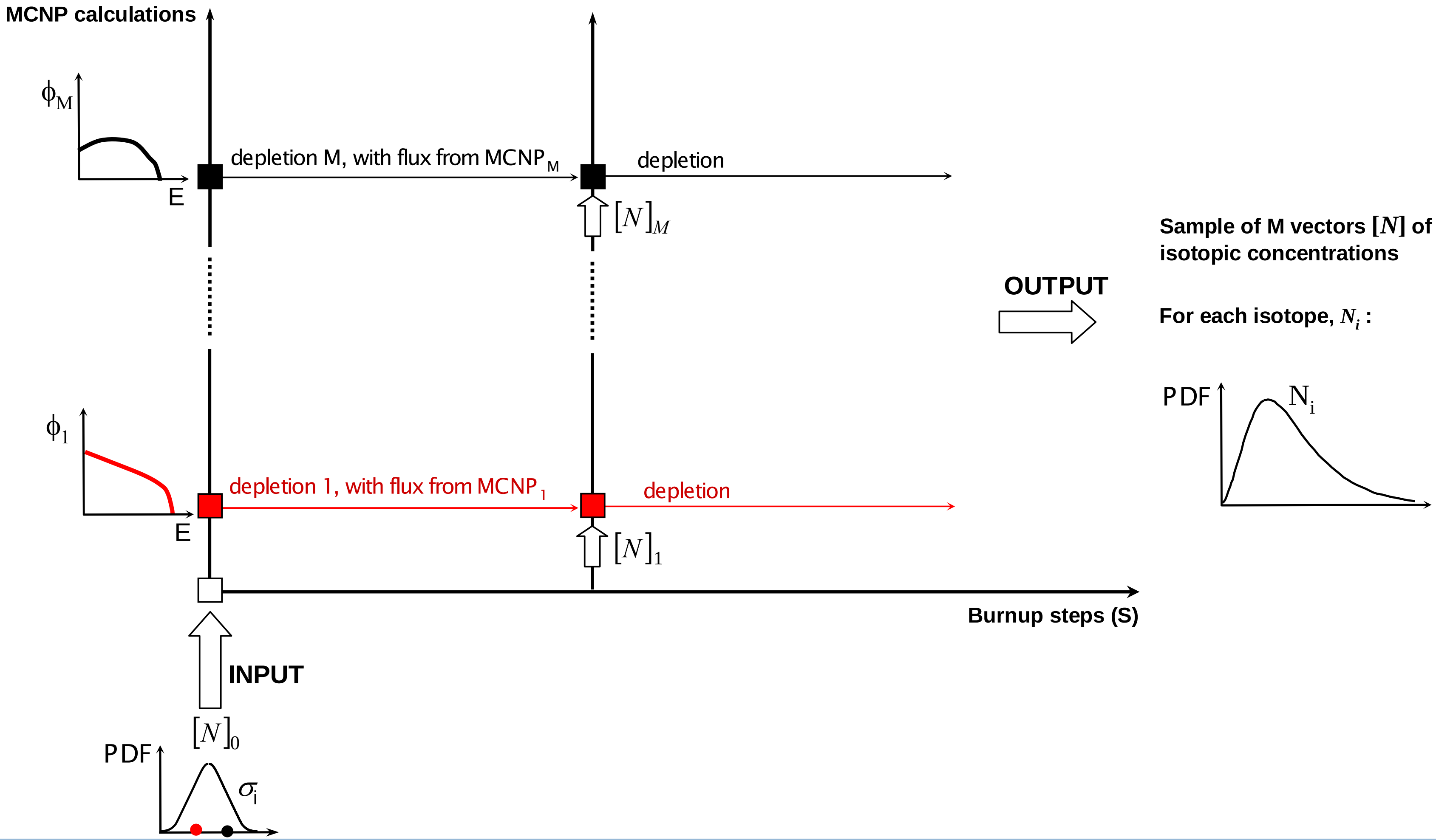
- Uncertainties in the integrated neutron flux: $\Delta\Phi$

Propagation of uncertainties in burn-up calculations

“Brute force” random sampling method

Same sequence that the coupled calculation scheme to infer an error propagation procedure throughout the time

Simultaneous random sampling of the PDF of all the input parameters



Propagation of uncertainties in burn-up calculations

Sensitivity/ Uncertainty Analysis (S/U)

Procedure based on a **first order Taylor** series approach

$$N_i(\sigma^{eff}) = N_i(\hat{\sigma}^{eff}) + \sum_{j=1}^{R} \left[\frac{\partial N_i}{\partial \sigma_j}\right]_{\hat{\sigma}^{eff}} \left(\sigma_j^{eff} - \hat{\sigma}_j^{eff}\right) + \ldots$$

Sensitivity coefficient ρ_{ij}

ε_j error in the 1-G effective xs

$$\sigma_j^{eff} = \sum_g \sigma_j^g \phi^g$$

$$\varepsilon_j = \sum_{g=1}^{G} \phi^g \left(\sigma_j^g - \hat{\sigma}_j^g\right) + \sum_{g=1}^{G} \sigma_j^g \left(\phi^g - \hat{\phi}^g\right) = \phi^T \boxed{\varepsilon_{\sigma_j}} + \sigma_j^T \boxed{\varepsilon_\phi}$$

errors due to uncertainties in the multigroup xs $\left[COV_{\sigma_j}\right]$

to be processed from the uncertainty libraries

errors due to uncertainties in the multigroup flux spectrum $\left[COV_\phi\right]$

to be obtained from a single MCNP calculation

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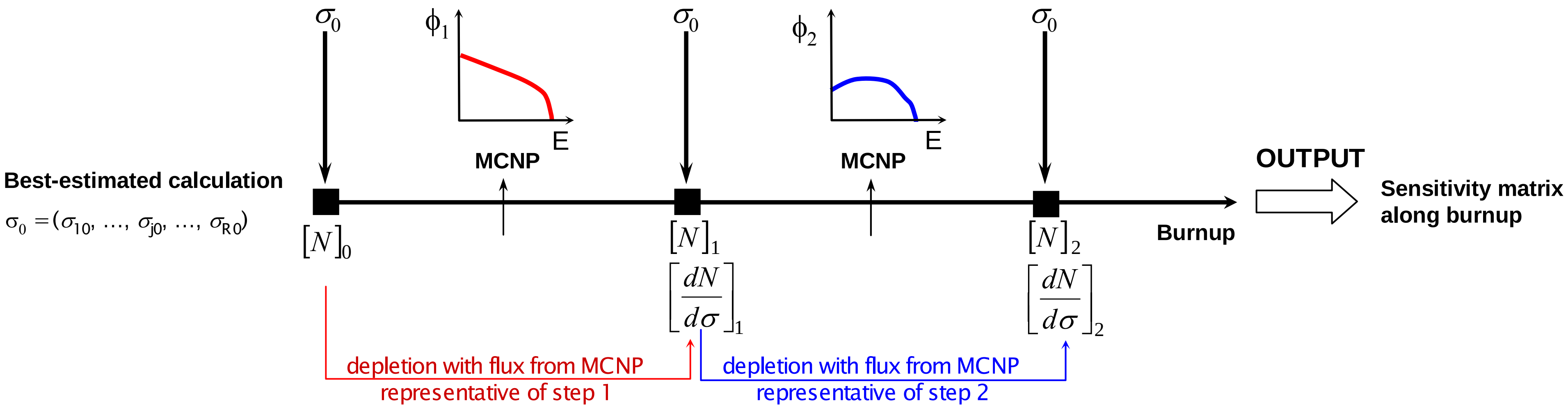
Propagation of uncertainties in burn-up calculations

Sensitivity/ Uncertainty Analysis (S/U)

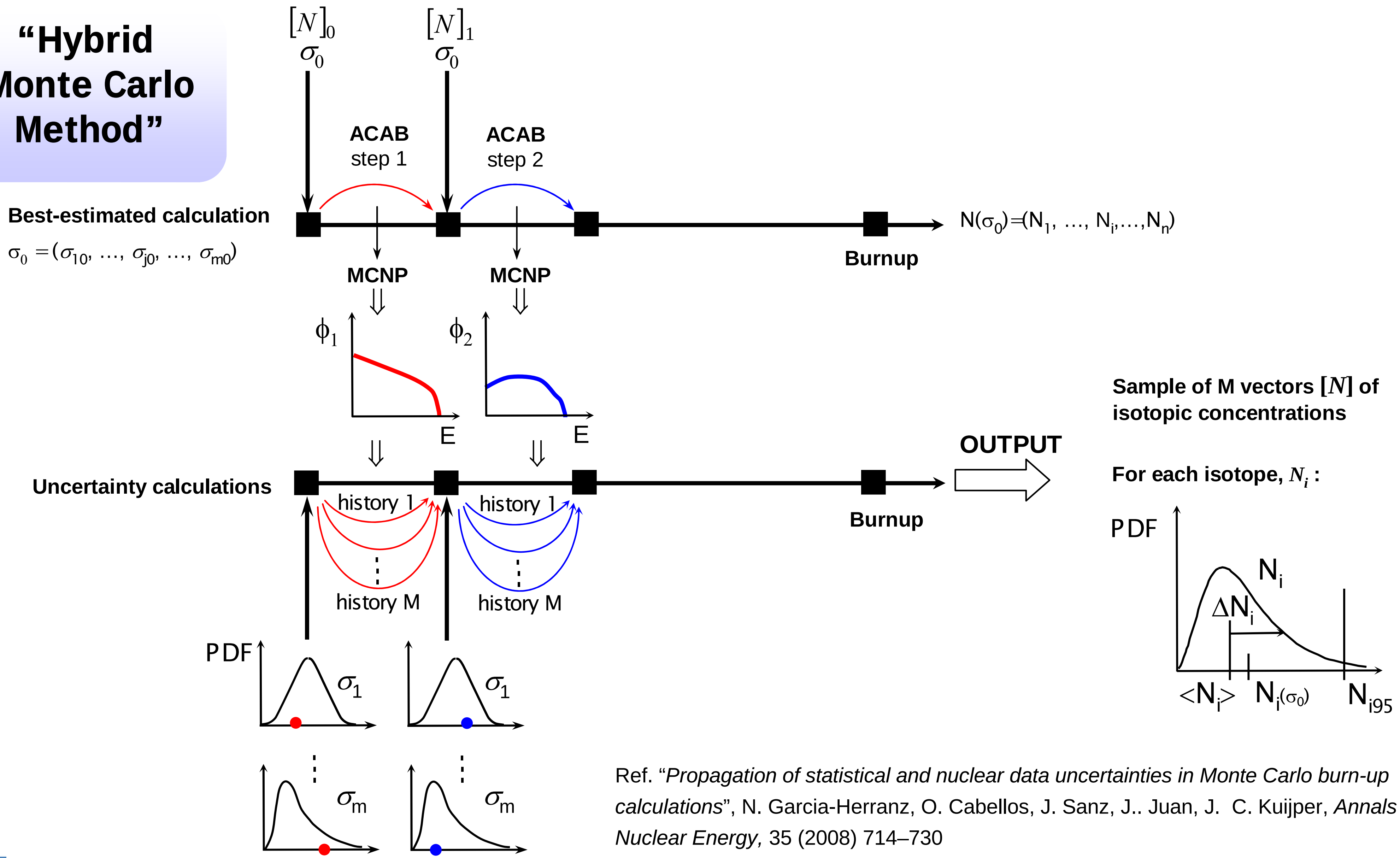
$$N\left(\sigma^{eff}\right) - N\left(\hat{\sigma}^{eff}\right) \approx \boldsymbol{S}\,\varepsilon$$

$$var\ N \approx \boldsymbol{S}\left[COV_{\sigma^{eff}}\right]\boldsymbol{S}^{\boldsymbol{T}} \approx \boldsymbol{S}\left\{ \underbrace{\begin{bmatrix} \ddots & & 0 & \\ 0 & & \hat{\phi}^{T}\left[COV_{\sigma_j}\right]\hat{\phi} & \\ & & & \ddots \end{bmatrix}}_{\text{Propagates the multigroup xs uncertainties when there is no statistical flux errors}} + \underbrace{\begin{bmatrix} \ddots & & 0 & \\ 0 & & \hat{\sigma}_j{}^{T}\left[COV_{\phi}\right]\hat{\sigma}_j & \\ & & & \ddots \end{bmatrix}}_{\text{Propagates statistical flux errors when there is no multigroup xs covariances}} \right\}\boldsymbol{S}^{\boldsymbol{T}}$$



Propagation of uncertainties in burn-up calculations

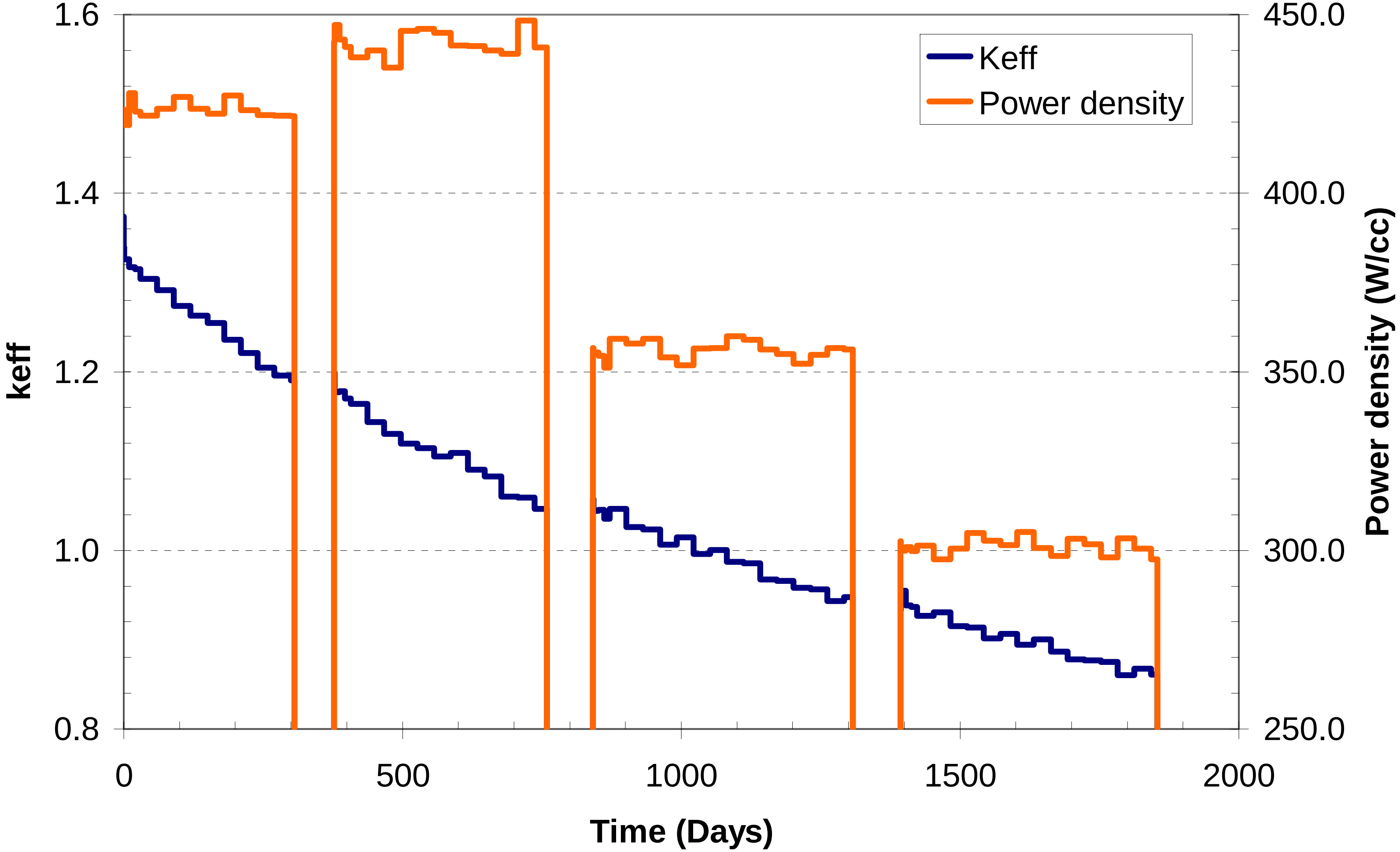
"Hybrid Monte Carlo Method"



Ref. "*Propagation of statistical and nuclear data uncertainties in Monte Carlo burn-up calculations*", N. Garcia-Herranz, O. Cabellos, J. Sanz, J.. Juan, J. C. Kuijper, *Annals of Nuclear Energy,* 35 (2008) 714–730

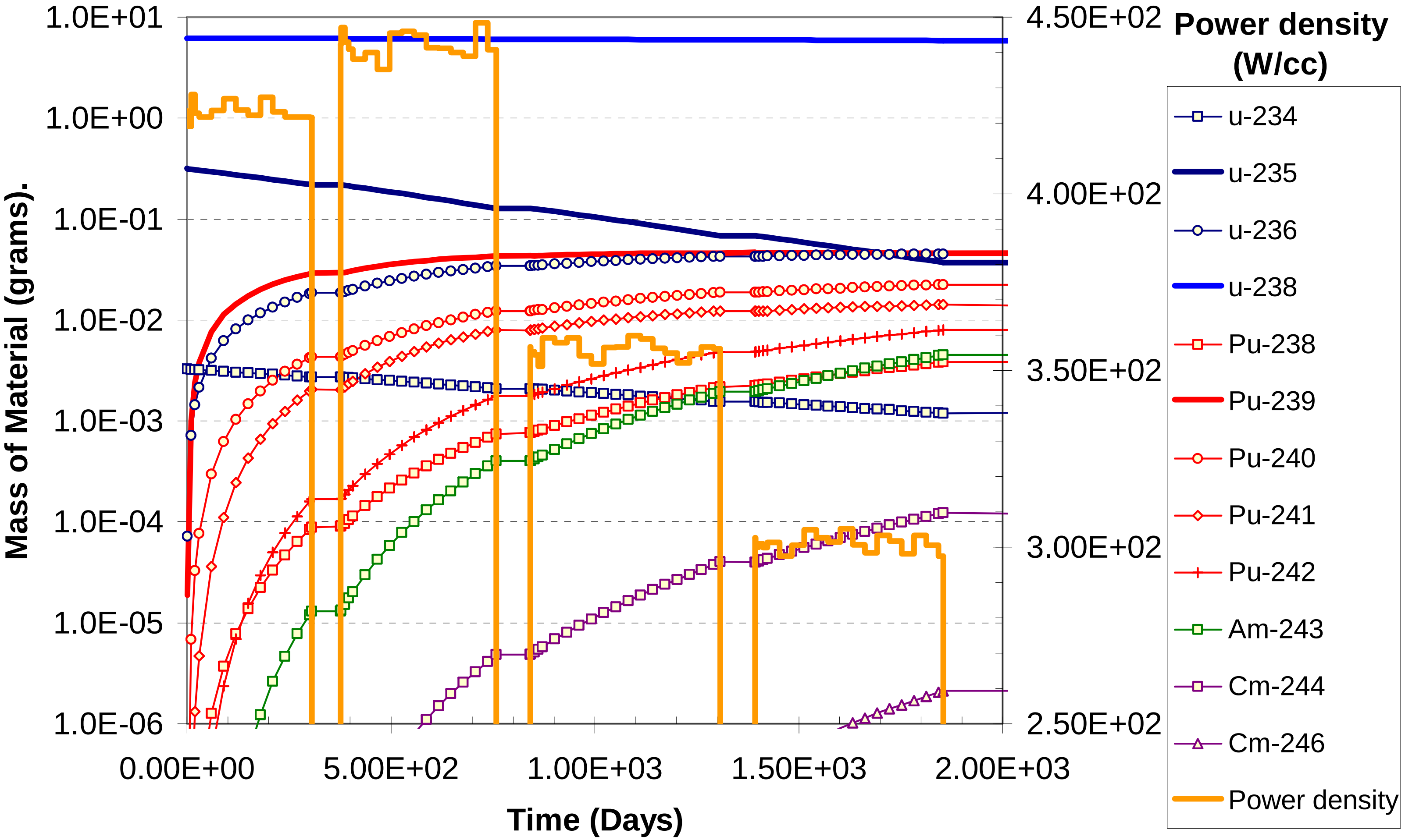
Application: PWR-HFP Exercise I-1

- PWR-HFP Burnup: 4 cycles taken from Phase-IB (case C). Burnup ~ 65 GWd/tMU
- Fluctuations in power density: transport-inventory coupling, time step, statistical errors, …



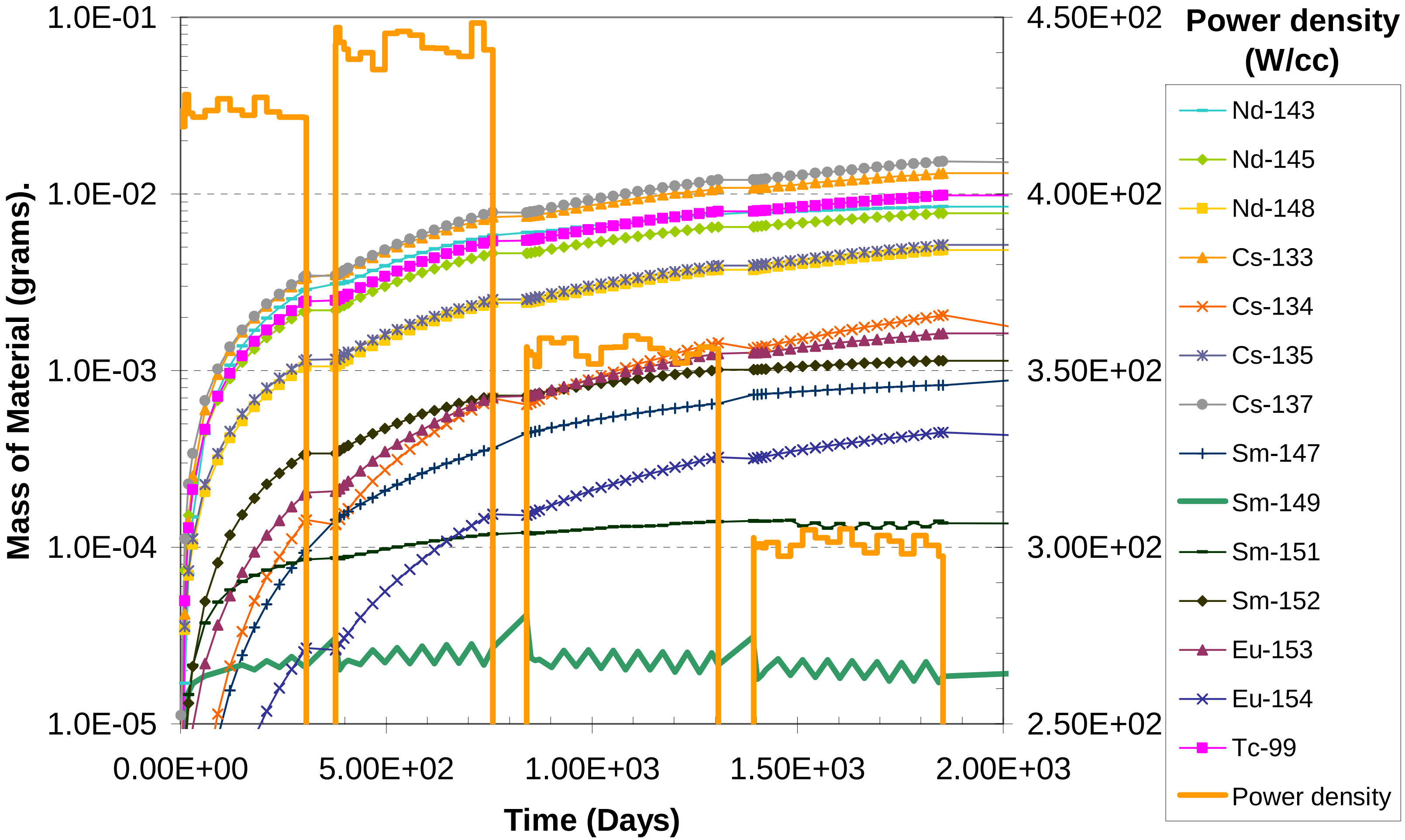
Iostopic predictions: Actinides

- Major and minor actinides versus burnup



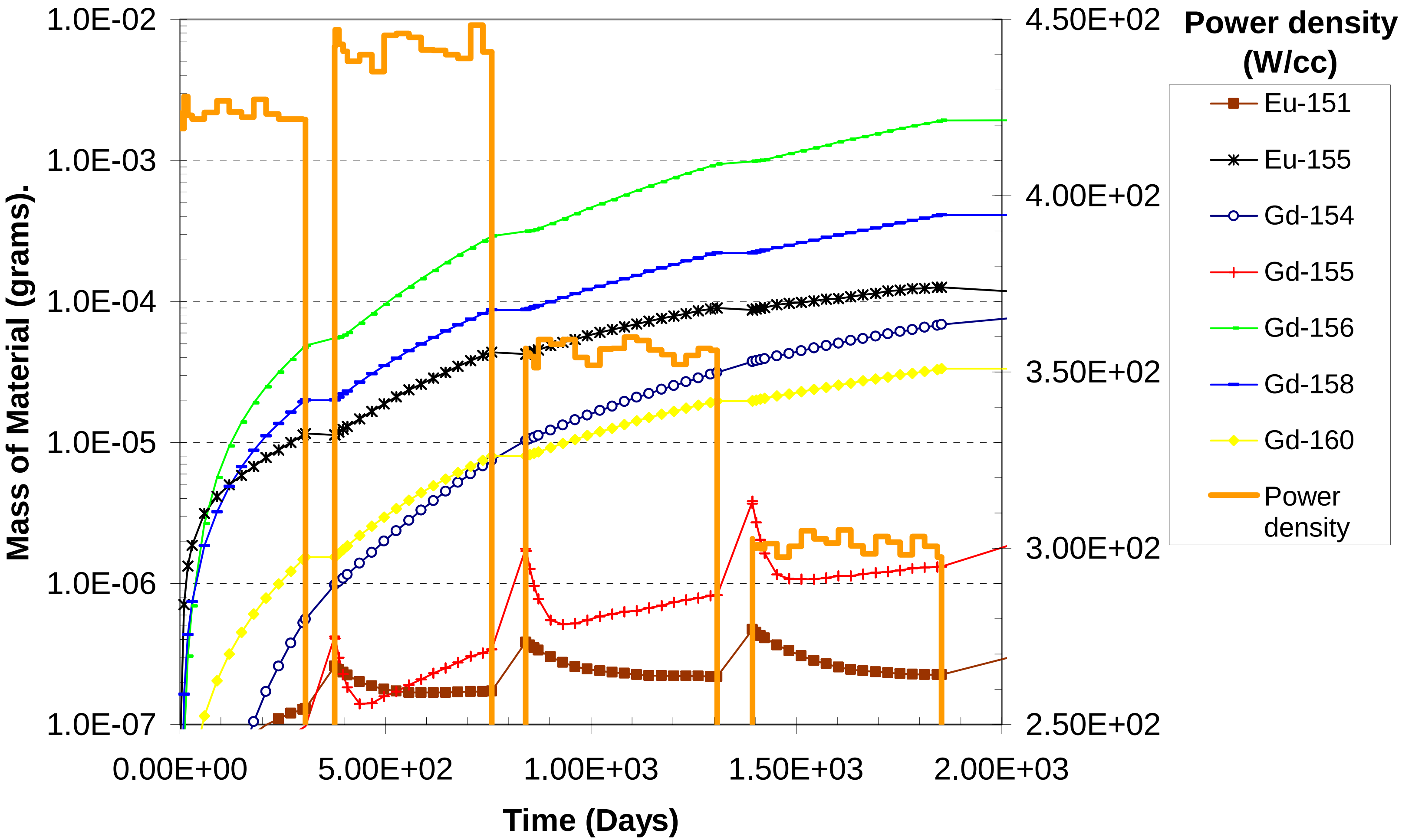
Iostopic predictions: Fission Products

- Fission Products versus burnup



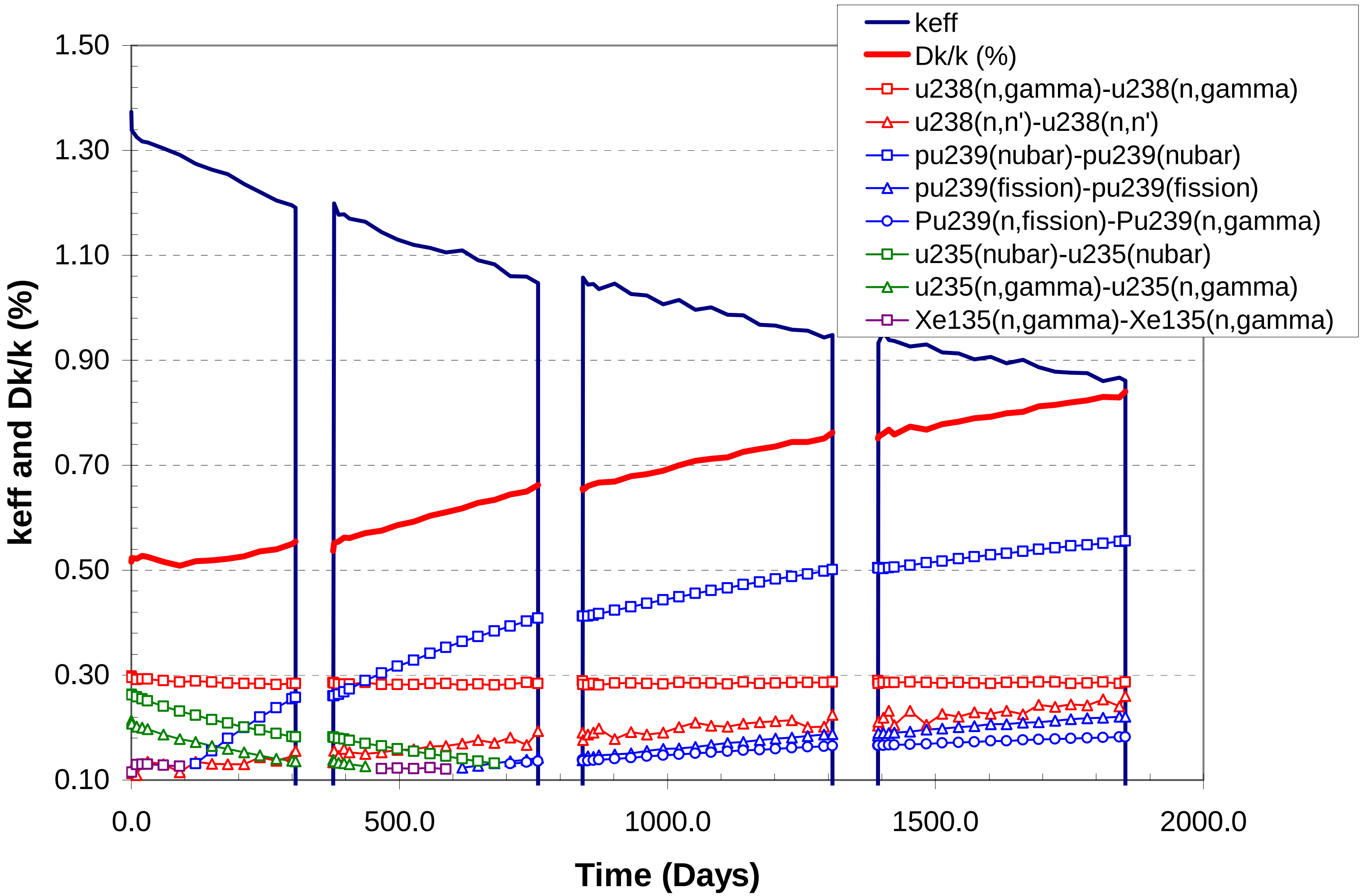
Iostopic predictions: Fission Products

- Fission Products versus burnup



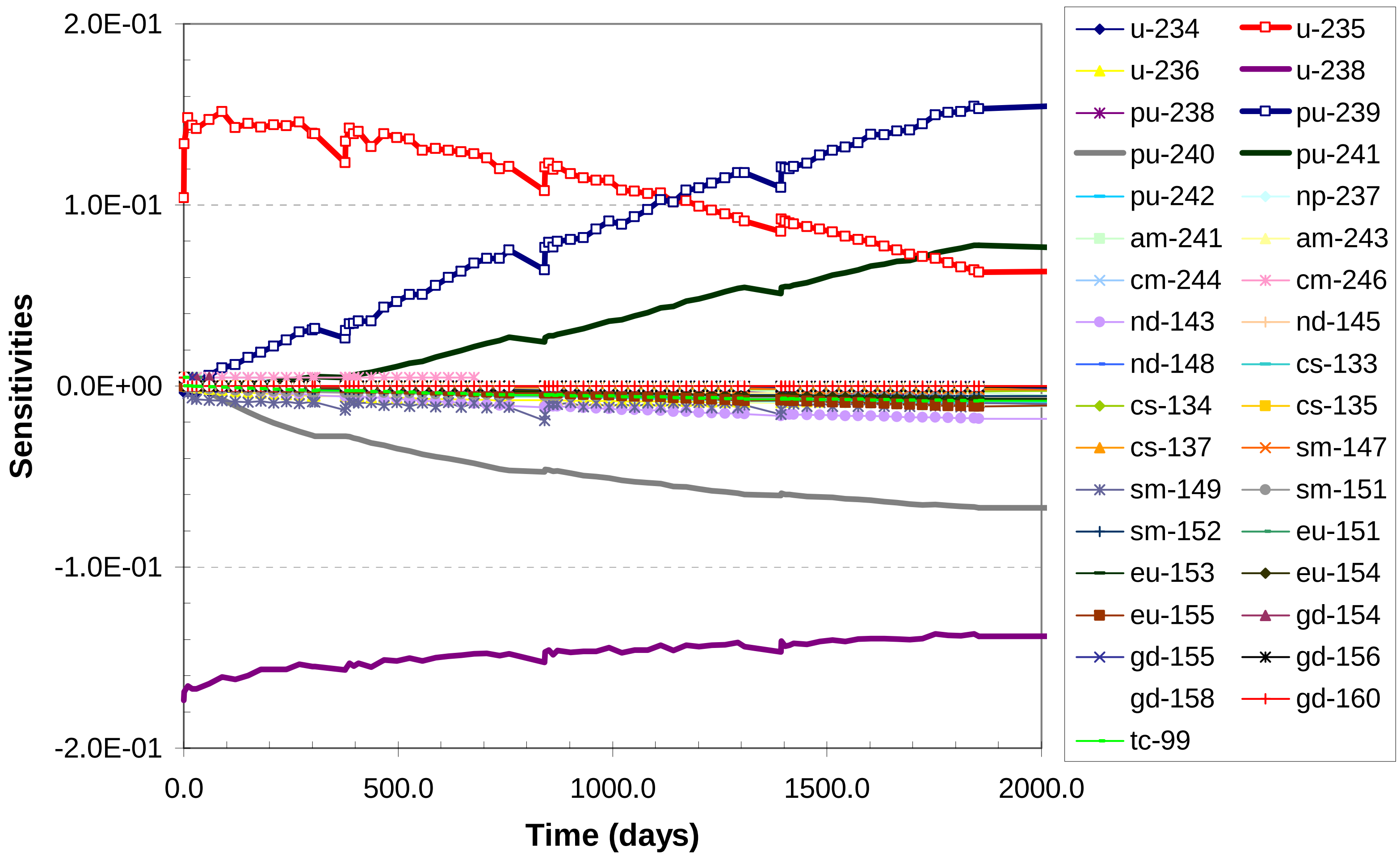
Prediction of Δk/k using TSUNAMI

- Δk/k (%) predicted with TSUNAMI (SCALE6.0) and the most important contributions
- Here, NO uncertainties in the isotopic inventory are taking into account



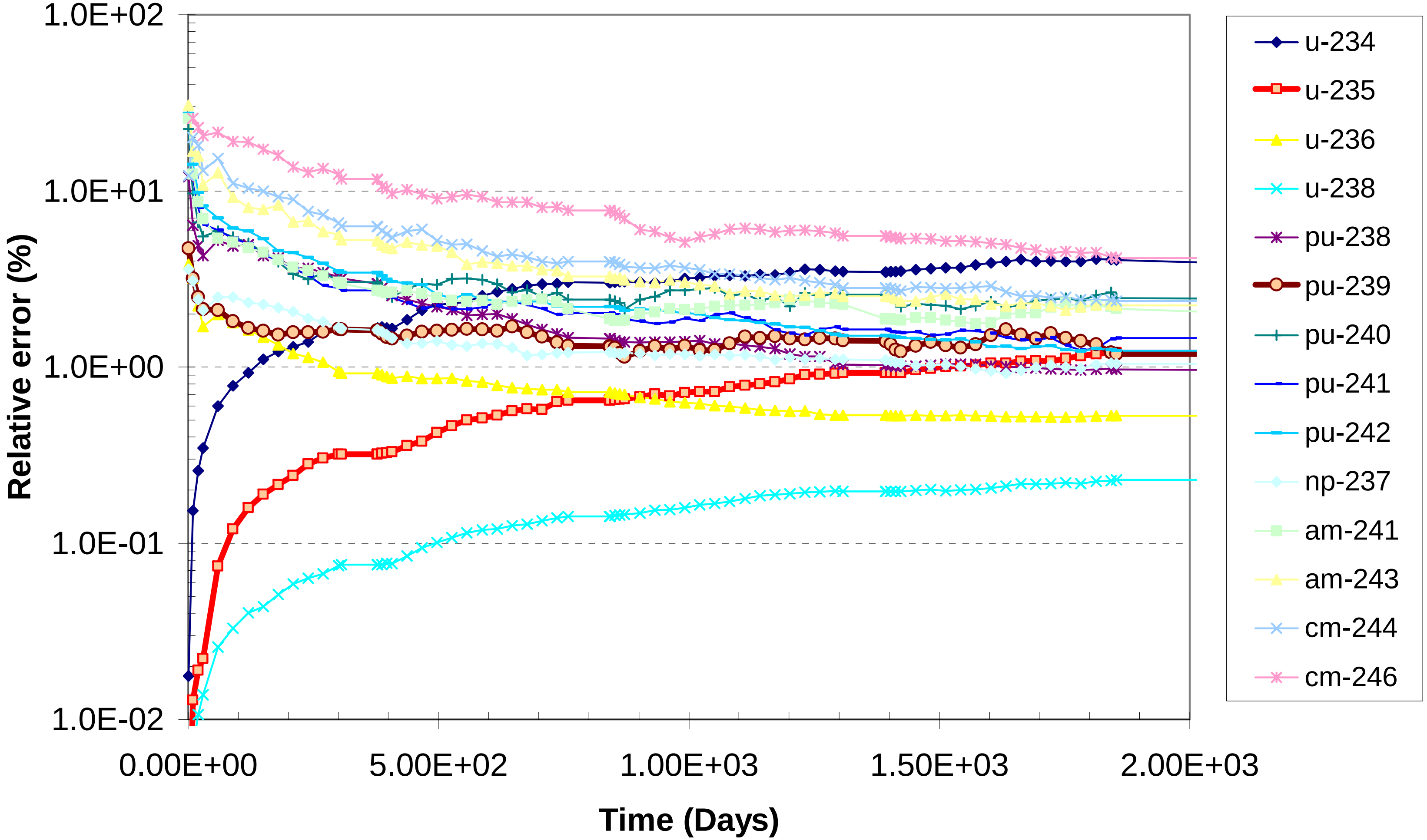
Prediction of sensitivities using TSUNAMI

- Sensitivitye (Δk/k/ΔN/N) predicted with TSUNAMI (SCALE6.0) and the most important contributions by isotopes



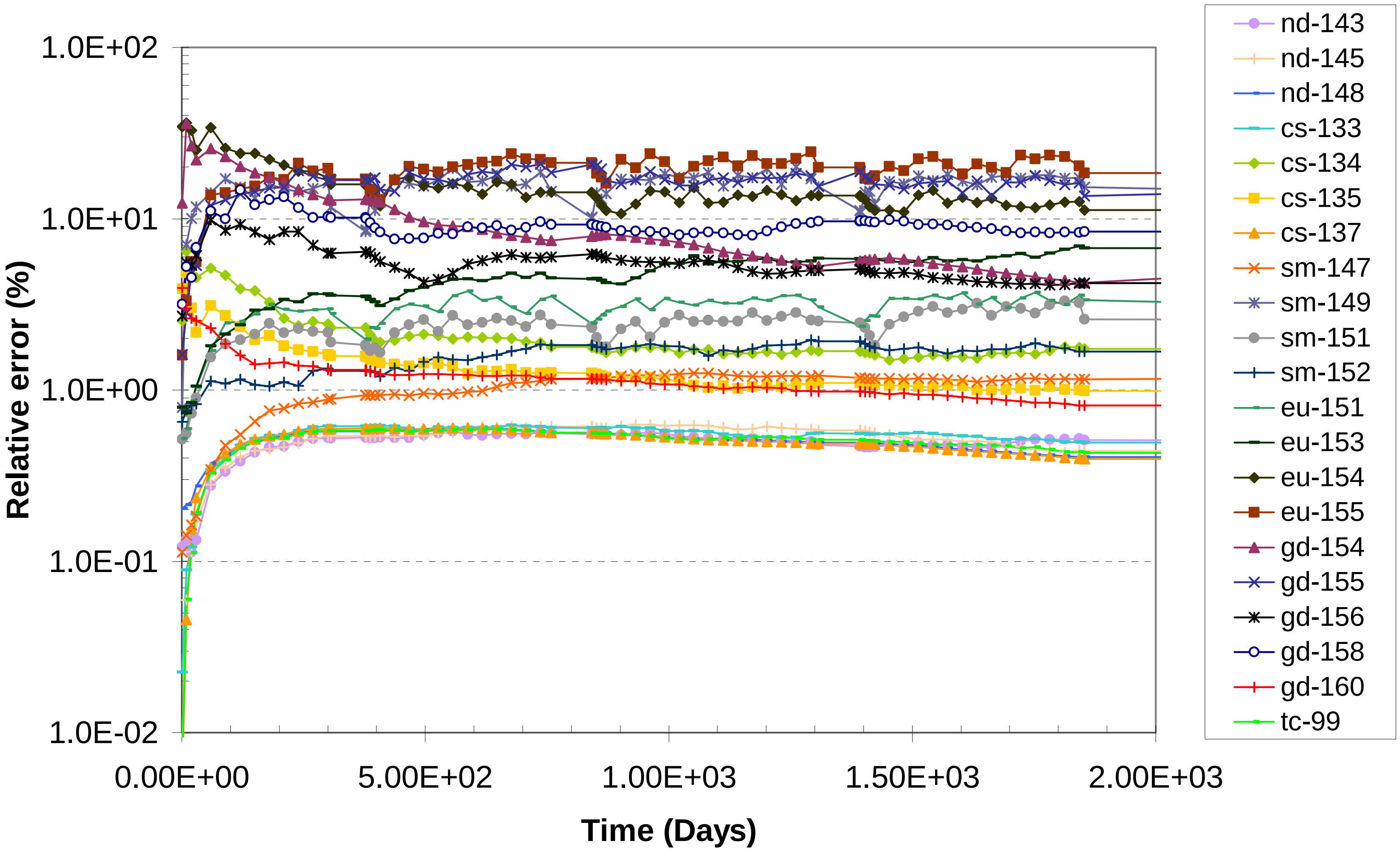
ΔN/N due to XS: Actinides

- ΔN/N (%) predicted with Hybrid Monte Carlo Method due to **uncertainties in XSs** (EAF2007/UN)



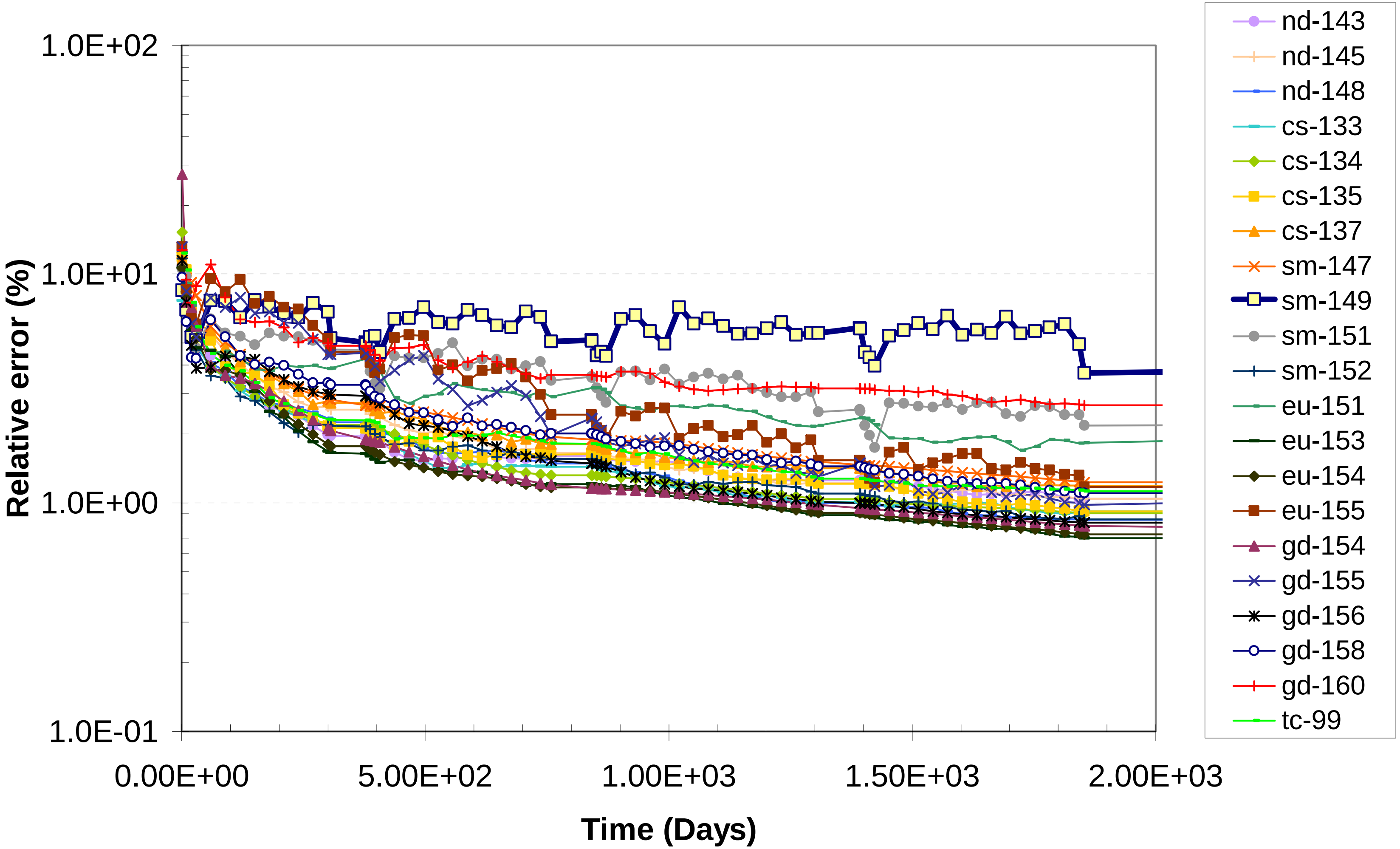
ΔN/N due to XS: Fission Products

- ΔN/N (%) predicted with Hybrid Monte Carlo Method due to **uncertainties in XSs** (EAF2007/UN)



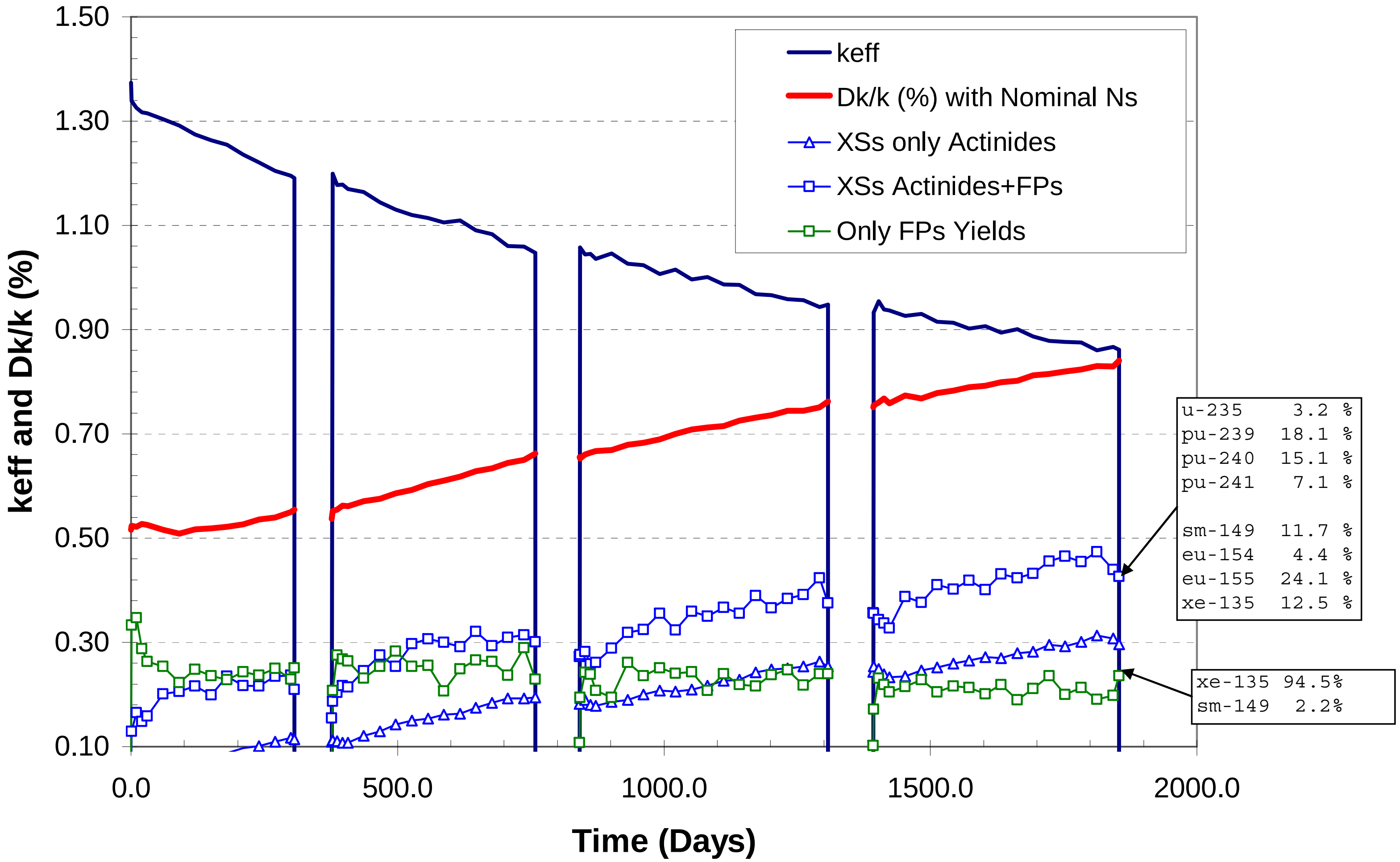
ΔN/N due to Fission Yields: Fiss. Produc.

- ΔN/N (%) predicted with Hybrid Monte Carlo Method due to **uncertainties in FYs** (JEFF-3.1.1)



Prediction of Δk/k due to ΔN/N

- Δk/k (%) due to the uncertainties in the isotopic inventory in comparison with the Δk/k (%) calculated by TSUNAMI/SCALE6.0 without isotopic inventory uncertainties
- Nuclear data uncertainties: cross-sections and fission-yields



In summary …

We have carried out a ***Burnup Uncertainty Analysis*** for the PWR-HFP Exercise I-1 Benchmark (Burnup ~ **65 GWd/tMU**)

1) Assuming **no uncertainties in the isotopic inventory**, TSUNAMI/SCALE6.0 predicts Δk/k (%) at BOC:~ 0.5% and EOC :~ 0.8%

   At EOC, the most important reactions are: Pu239(nubar), U238(n,gamma), U238(n,n'), Pu239(fission) and Pu239(fission-capture)

2) To take into account **uncertainties in the isotopic inventory**, an Hybrid Monte-Carlo methodology that links transport and inventory calculations is presented

   It enables to estimate the impact of nuclear data (neutron cross section and fission yields) uncertainties on the inventory in transport-burnup combined problems.

   At EOC, we predict the values of Δk/k (%) due to ΔN/N:

   - Cross-sections for actinides:~ 0.3% and for fission products :~ 0.2%
     The most important isotopes:Pu239 and P240; Eu-155, Xe135 and Sm149

   - Fission yields: :~ 0.2%
     The most important isotopes: Xe135